B.Sc. - I (NEP) Semester-II

BSCMA601 - Mathematics DSC : Integral Calculus

P. Pages : 3 **GUG/S/25/16769**

Time : Two Hours Max. Marks : 80

Notes : 1. Solve all the five questions.
2. First four questions carry equal 15 marks and the fifth question carry 20 marks.
3. The use of calculator is not permitted.

UNIT – I

1. Solve **any three**.

a) If $x = \phi(t)$ is a differentiable function of t then prove that: **5**

$\int f(x)\,dx = \int f(\phi(t)).\phi'(t)\,dt$

And hence evaluate $\int 3x^2.\sin(x^3)\,dx$

b) Evaluate $\int x.\tan^{-1} x\ dx$. **5**

c) Evaluate $\int \frac{2x^2-3}{(x^2-5)(x^2+4)}dx$. **5**

d) If $I_n = \int \cos^n x\ dx$ then prove that $I_n = \frac{1}{n}\cos^{n-1} x.\ \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$. **5**

e) Evaluate **5**

i) $\int_0^{\frac{\pi}{4}} \sin^4 x\,dx$

ii) $\int_0^{\frac{\pi}{8}} \cos^3 2x\,dx$

UNIT – II

2. Solve **any three**.

a) Prove that $\int_{-a}^{a} f(x)\ dx = 0$,f(x) is an odd function **5**

$= 2\int_0^a f(x)dx,\ f(x)$ is an even function.

b) Show that $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\ dx = \frac{\pi}{4}$. **5**

c) Find $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$. **5**

d) Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where $a > 0$. **5**

e) Find the area of the region included between $y^2 = 2x$ and the line y = 2x. **5**

UNIT – III

3. Solve **any three**.

a) Show that refinement of a partition P increases lower sums i.e. $L(P, f) \le L(P^*, f)$. **5**

b) Show that any constant function defined on a bounded closed interval is integrable. **5**

c) If f is a bounded and integrable over [a, b] and M, m are bounds of f over [a, b] then prove that $m(b-a) \le \int_a^b f(x)\, dx \le M(b-a)$. **5**

d) Prove that every continuous function is integrable. **5**

e) If f is continuous on [a, b] and F is continuous and differentiable on [a, b] with $F'(x) = f(x), x \in [a, b]$ then prove that $\int_a^b f(x) dx = F(b) - F(a)$. **5**

UNIT – IV

4. Solve **any three**.

a) Prove that $\int_2^{\infty} \frac{x^2}{\sqrt{x^7+1}} dx$ converges but $\int_2^{\infty} \frac{x^3}{\sqrt{x^7+1}} dx = \infty$ **5**

b) Show that the integrals i) $\int_0^{\infty} \frac{\cos x}{\sqrt{1+x^3}} dx$ and ii) $\int_1^{\infty} \frac{x}{3x^4+5x^2+1} dx$ converges absolutely. **5**

c) Prove that $\int_0^{\infty} x^{n-1} e^{-kx} dx = \frac{\Gamma n}{k^n}$, where n, k > 0 are constants and hence evaluate $\int_0^{\infty} x^{1/4} e^{-\sqrt{x}} dx$. **5**

d) Prove that $B(m, n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta . \cos^{2n-1}\theta d\theta$. **5**

e) Prove that $\Gamma 2m = \frac{2^{2m-1}}{\sqrt{\pi}} \Gamma m \, \Gamma\left(m+\frac{1}{2}\right)$. **5**

5. Solve **any ten**.

a) Evaluate $\int \frac{\cot(\log x)}{x}\,dx$. **2**

b) Evaluate $\int \sin x . x\,dx$. **2**

c) Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x . \cos^4 x\,dx$. **2**

d) Prove that $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$. **2**

e) Evaluate $\int_0^1 x(1-x)^{1/2} dx$. **2**

f) Find the area bounded by the curve $y = x^2$, the Y-axis, the X-axis and x = 3. **2**

g) Define norm or mesh of a partition. **2**

h) For any partition P, prove that $L(P,f) \leq U(P,f)$. **2**

i) Prove that $\int_1^{-1} \cos^8 x dx < 0$. **2**

j) Define improper integral. **2**

k) Test the convergence of $\int_1^{\infty} \frac{dx}{x^3}$ **2**

l) Prove that $\Gamma 1 = 1$. **2**

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